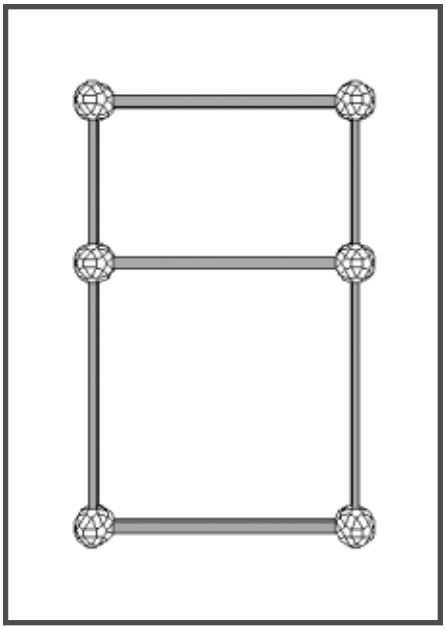
Zome System

Builds Genius!

Similarity and the Golden Section

Mathematics
Advanced Concept

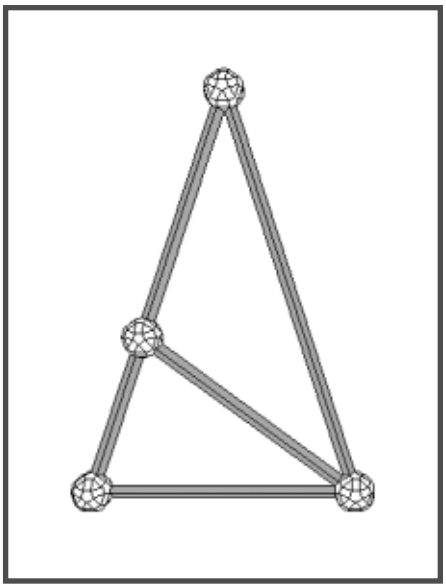


Lesson Objective:
Students will learn how the algebraic relationships contained in the golden section can be directly derived from a geometric relationship called similarity.

Prerequisite Skills:
Knowledge of the quadratic equation. Some background on the Golden Section ("Finding Tau," and "The Golden Thread-A History of Tau").

Time Needed:
One or two class periods of 45-60 minutes.

Materials Needed:
• One Zome System Creator Kit for 25-30 students



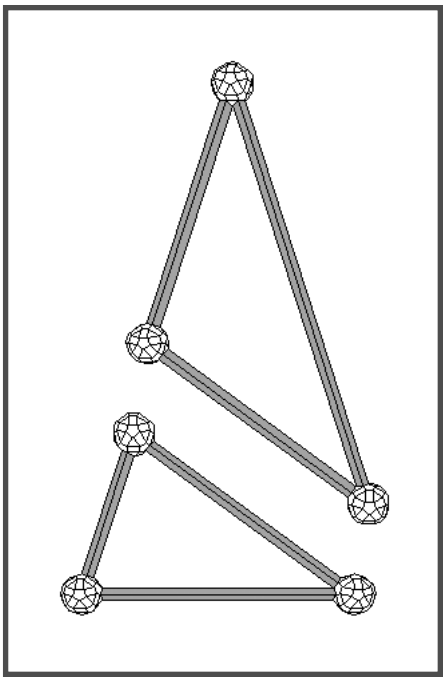
Procedure:
Prepare by drawing on the board the two shapes shown in the graphic. Indicate only the 108º angle of the triangle. Introduce the class by informing the students that they are going to algebraically derive the Golden Section using the concept of similarity. Direct their attention to the shapes on the board. *What are the shapes? How are they divided? Is there a relationship between each shape and its division?* Working in pairs, students should build Zome System models of the two shapes, using blue struts. They should also draw the triangle on paper and label the sides and angles. If it is given that the length of the long segment of the divided leg is x, and the short segment is 1, they should label the other sides using deduction. Allow the students a few minutes to complete this, and then go through the deduction with the whole class, as follows.

First, from the given information, the full length of the divided leg is X+1. Therefore, the other long leg is X+1 because they are the two similar legs of an isosceles trian-

gle. Second, the inner line is the same length as the upper divided section since the upper triangle is also an isosceles triangle, so its length is X. And finally, the base of the whole figure must be the same as the inner line since the lower triangle is also an isosceles triangle, so its length must be X.

Ensure that each student follows this reasoning, and has labeled all sides correctly, as shown.

Next, have the students rebuild the triangles, separated for clarity as shown.



Now repeat the entire process using the divided rectangle. Review with the class how we can proportion from ratios for both figures as shown.

$$\frac{1}{X} = \frac{X}{X+1}$$

Cross multiplying, we get

$$X^2 = X + 1$$

Give the pairs of students ten minutes to try to reach the Golden Section, via the quadratic equation:

$$X = \frac{1 + -\sqrt{5}}{2}$$

Finally, go through each step with the entire class.

As an extension, discuss the relevance of the golden section in mathematics and fine art.

Assessment:

Question students about their deduction process, and review notes in their math journals. To meet the standard students must correctly label the lengths of the triangles and the rectangles, and follow the derivation of the golden section. To exceed the standard they must derive the section on their own.

Standards Addressed:

* Mathematics standards addressing the continued study of algebraic concepts and methods (NCTM Standard 5).
* Mathematics standards addressing the continued study of the geometry of two and three dimensions (NCTM Standard 7).

Transfer Possibilities:

Doing trigonometry and algebraic simplification with the Golden Section (pages 21-24 in “Zome System Manual”).