

Mathematics Intermediate Concept

Lesson Objective:

Students will learn about three prime factors through deductive reasoning.

Prerequisite Skills:

Familiarity with concept of prime numbers. Students should also have worked on relating geometric shapes to numbers (“Shape and Number”).

Time Needed:

One class period of 45-60 minutes.

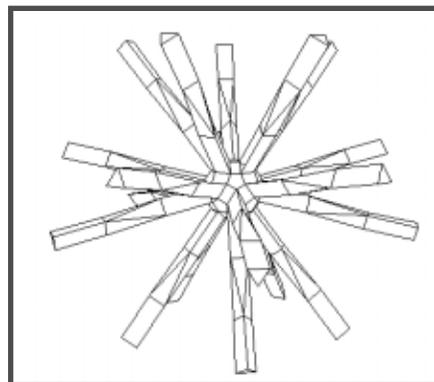
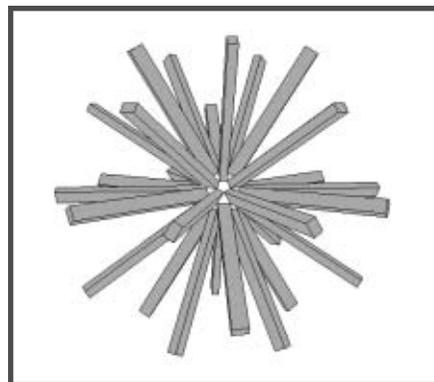
Materials Needed:

- One or two Zome System Creator Kits for class of 25-30 students.
- Standard (analog) clock

Procedure:

Divide your class into teams of 4 students. Give each team 4 Zome System nodes and struts in one length in all 3 colors. For example, one group will get 4 nodes and short red, yellow, and blue struts. It is best to use short and medium struts, although the long will also work, if the class is very large.

Hold a brief discussion of the Zome System node. *How many different shapes of holes are there in the nodes? What are the shapes? How many holes are in each node? What would be an effective way of counting the number of holes?* Let the teams start this problem solving exercise. Several of the teams will divide the work into counting rectangular holes, triangular holes and pentagonal holes. They will find all of one particular kind of hole by building a “pincushion;” that is, putting one color strut in every corresponding hole in the node. Encourage struggling teams to adapt the pincushion method.



Prime Factors

Zome System

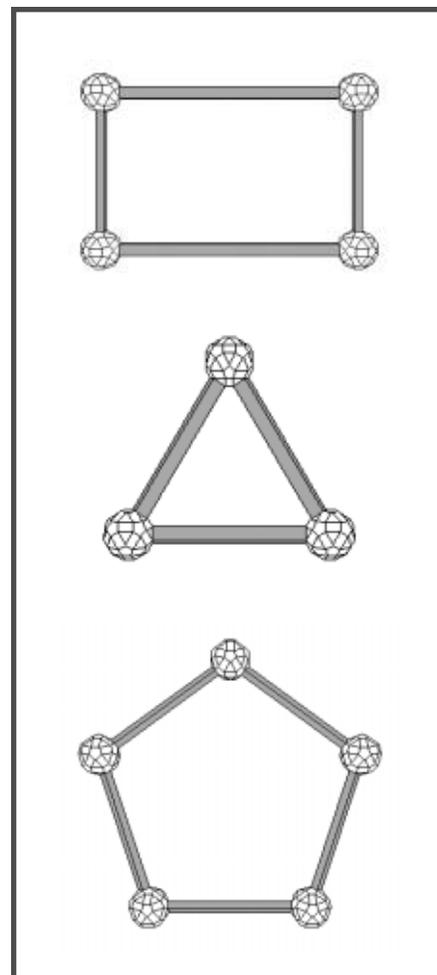
Builds Genius!

Allow 5-10 minutes for students to build pincushions and count the number of struts. *What's the best way to count the struts?* While many students will want to remove the struts and count them, others will have discovered that when the pincushion is flat on a desk top, the struts can be counted using symmetry. For example, a yellow pincushion has 4 "levels," each containing 5 struts. It is now easy to see that there are 20 triangular holes. The blue pincushion is especially interesting since it can stand on either 3 or 5 struts.

Chart the students' results on the board.

Is there a relationship between the shape of the hole and the number of holes in the node? In the lesson "Shape and Number Students" students established that the rectangular hole is the number 2, the triangular hole is the number 3 and the pentagonal hole is the number 5. Lead your students to multiply the "shape" by the number of holes in the node. Students will discover that the result is always 60.

How can we get the number 60 from the numbers 2, 3 and 5? Lead your students to prime factors. *Are there any smaller whole numbers which multiply together to make 60?* Let your students explain why the number 1 doesn't count. *Where else can we find the number 60 in this classroom?* Someone will discover the clock. *Why are there 60 minutes in an hour?* Lead students to find how many ways 60 minutes can be



Shape	Number represented by shape	Holes per level	Number of levels	Subtotal	total holes
rectangle	2	5	4	20	30
		10	1	10	
triangle	3	5	4	20	20
pentagon	5	3	4	12	12
				grand total	62

divided into whole number increments (2, 3, 4, 5, 6, 10, 12, 15, 20, 30...) Ask the class to find other familiar numbers with 2, 3 and 5 as prime factors (24 hours in a day, 360 degrees in a circle)

As an extension activity students can explore how many lines pass through each node. Lead students to discover that each strut in the pincushion represents a vector originating at the point (node) in the center. Two 2 vectors (struts) point in opposite directions to form a line in space. The relationship between the shape of the strut (it's number) and the number of such lines passing through the node can be described with the diagram on page 106. To obtain the number of red, or number five lines, cover the 5 and multiply the remaining numbers ($3 \times 2 = 6$). As each line consists of two strut there are then 12 red holes in the node. Test that the system holds up for the other colors.

Assessment:

Observe students while they work, and question them about the method they used to count the number of holes in the node. To meet the standard students must use the symmetry method to determine that the nodes have 62 holes. To exceed the standard they must identify the number 2, 3, and 5 as being prime factors for 60.

Standards Addressed:

- * Mathematics standards addressing **mathematical problem solving as a method of inquiry and application** (NCTM Standard 1).
- * Mathematics standards addressing **number systems and number theory** (NCTM Standard 6).
- * Mathematics standards addressing **the study of the geometry of one, two, and three dimensions** in a variety of situations (NCTM Standard 12).

Transfer Possibilities:

Prime numbers and prime factors in common experience.

1	2	3	5	7	11	13	17
19	23	29	31	37	41		
43	47	53	57	59....			