

Mathematics Intermediate Concept

Lesson Objective:

Students will start their exploration of regular polyhedra shapes. Based on how many polygon faces can meet at a vertex of a solid, they will determine that there only can exist five such 3-dimensional shapes.

Prerequisite Skills:

Knowledge of basic polygons ("Geometric Shapes"), and ability to define a two-dimensional versus a three-dimensional figure ("2-D and 3-D Shapes").

Time Needed:

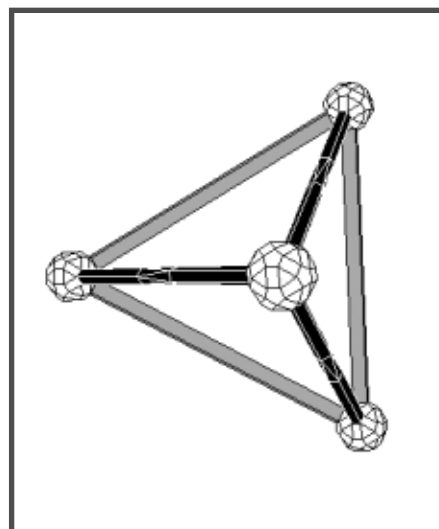
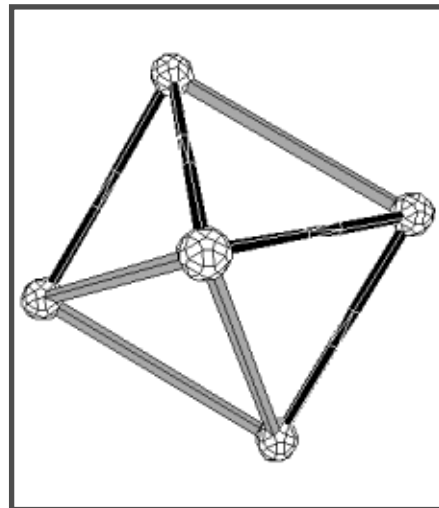
One or two class periods of 45 to 60 minutes.

Materials Needed:

- Two Zome System Creator Kits for 25-30 students
- Three or four packs of supplementary green Zome System struts, if available
- One pair of scissors per team
- One roll of tape per team
- Card stock polygons. See "Resource" section in this document.

Procedure:

Start the class with a brief review of polygons and polyhedra. *What is a polygon? What do their names tell us about them? What is a **regular polygon** (all vertices and angles are identical)? What is a **polyhedron**? Are polyhedra always made up of polygons? How are polyhedra named?* The present class will cover a special type of polyhedra called regular polyhedra. These are solids which have only regular polygons for faces and only one kind of face. The same number of faces meet in each vertex. Show a cube build with Zome System (blue struts) as an example of a regular solid consisting of square faces. All faces are squares, and three



Plato's Solids - I

Zome System

Builds Genius!

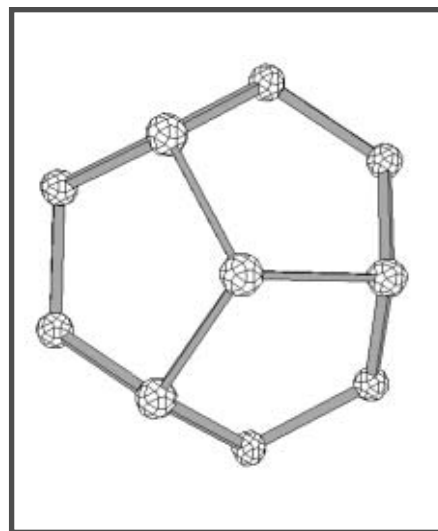
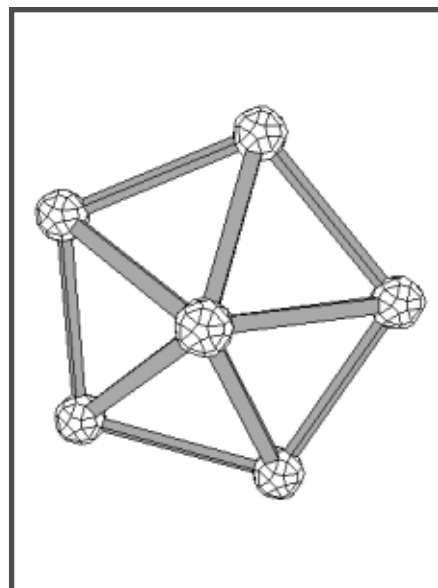
faces meet in each vertex.

Divide the class into teams and distribute the Zome System pieces, the scissors, tape, and card stock polygons. The task for the teams is to discover how many regular polyhedra there are. *How should we go about determining how many regular polyhedra exist? Is the number unlimited or are there a set number?* Ask the students to write down their predictions in their math journals. Discuss the various strategies that the students propose.

Let the students build for 20-30 minutes trying to create as many regular solids as possible. Discuss each solid the students offer as a solution. *Are all the faces the same polygon? Do the same number of faces meet in each vertex?* In this initial exploration it is acceptable to have solids where the faces are not regular polygons. How many did the students find?

Next students should focus on examining the vertices one at a time, using the various regular polygons, starting with the simplest. *What is the simplest regular polygon (the triangle)? Can there be different numbers of edges going into a vertex? What about the angles of the vertices? Can they vary? How many will make a vertex? Do two triangles make a full vertex? Do three? The students should attempt to build each vertex in turn. If you have the supplementary green struts, you will be able to make all the vertices with regular triangles. If you do not, you must use a combination of red and blue struts to complete the vertex consisting of three triangles and the vertex consisting of four triangles. Tape the cardstock triangles together at the edges to reinforce how the triangles fit together. What is the maximum number of triangle that will fit into a vertex (Five is the maximum. Six triangles will form a plane hexagon, thus not a vertex.)?*

When the limit is reached for triangles, move on to the next shape. *What is the next regular polygon? How many squares fit together to make a vertex? How many ways can squares make vertices? How many will make a flat fit? Again, make models in Zome System of the vertices that work. What is the next regular polygon? How many fit together into a vertex? Continue this until the teams discover the limits.*



Zome System

Builds Genius!

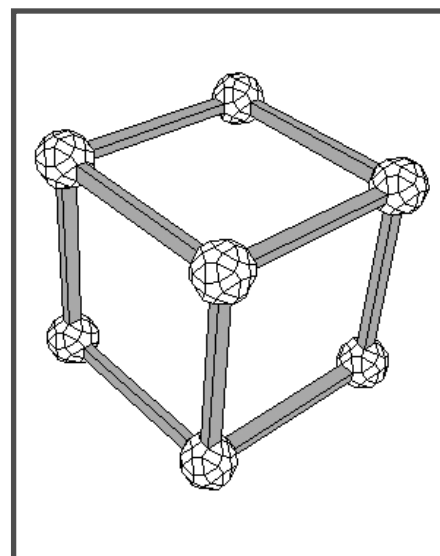
Plato's Solids - I

Hexagons fit together in a flat configuration only, and beyond six sides, the angles at the vertices are too big ($<120^\circ$) to put more than three together.

How many vertices have been found? Looking at just the vertices, can we tell what the completed solid will look like? Students should write down their observations and attempt to formulate a general rule describing the three-dimensional vertices that can be created using regular polygons. The teams should save the both the paper and the Zome System vertices they have built.

Regular solids have been studied all over the world for a very long time. The Greek philosopher Plato codified these forms around 2,400 years ago. He stated that only five solids could be created by repeating a regular two-dimensional polygon to complete a three-dimensional polyhedron. The criteria he used was that all features of the solids had to be regular, identical faces, angles, and vertices, without any overlap or gaps.

The class "Plato's Solids - II" continues the exploration of these polyhedra.



Assessment:

Observe and students as they build their structures, and take notes of their findings. Review notes in math journals.

To meet the standard, students must build both paper and Zome System vertices and determine how many are possible. To exceed the standard they must also formulate a rule which states why only five vertices are possible.

Standards Addressed:

- * Mathematics standards addressing **mathematical problem solving as a method of inquiry and application** (NCTM Standard 1).
- * Mathematics standards addressing **mathematics as reasoning** (NCTM Standard 3).
- * Mathematics standards addressing **the study of the geometry of one, two, and three dimensions** in a variety of situations (NCTM Standard 12).

Transfer Possibilities:

Continued exploration of Polyhedra ("Plato's Solids - II", "Archimedean Solids", and constructions 4, 5, 6, and 8 in Zome System Manual). More work on three-dimensional tessellations ("3-D Triangle Tiles" and "Beehive City").