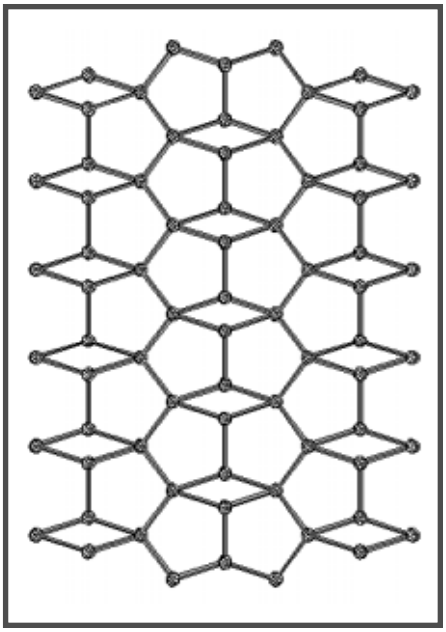
Zome System

Builds Genius!

Kepler's Tilings

Mathematics / Art
Intermediate Concept

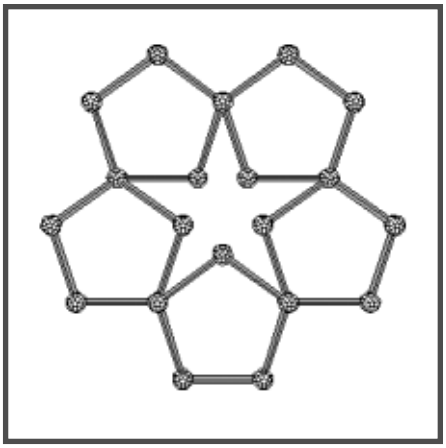
Lesson Objective:
Students will learn that while regular pentagons can not tile a plane by themselves, they can tile with a few other shapes. Students will identify these tilings as being non-periodic.



Prerequisite Skills:
Some previous work with tessellation ("Trying Tessellation," "What are Quadrilaterals" "Tiling with Quadrilaterals," and "Plane Patterns"). Basic understanding of symmetry concepts ("What is Reflection Symmetry?" "Multiple Reflection Symmetry," "Rotational Symmetry." and "Translational Symmetries in Tilings").

Time Needed:
Two class periods of 45-60 minutes.

Materials Needed:
- Two Zome System Creator Kits for 25-30 students



Procedure:
Review with the students the tilings created in the previous lesson. *What is special about the repetition in those tilings? How many different shapes were used in one tiling?* When one or several shapes are repeated in a continuos pattern, the tiling is called periodic. When the pattern changes or is interrupted, the tiling is called non-periodic. People around the world have worked with tilings and mosaics of many kinds. The German scientist and astronomer Johannes Kepler (1571-1630), who explored non-periodic tilings extensively. The tilings in this lesson are often known as Kepler Tilings.

Divide the students into teams, and distribute the Zome System pieces. The teams should build simple periodic

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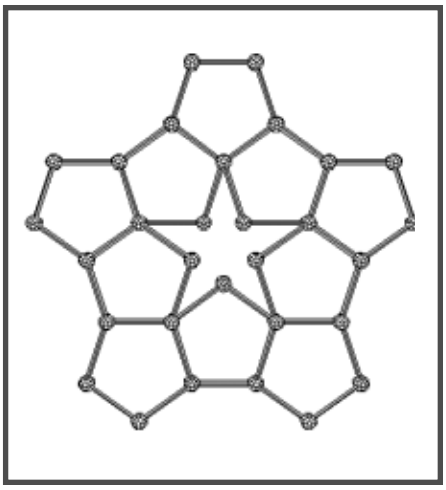
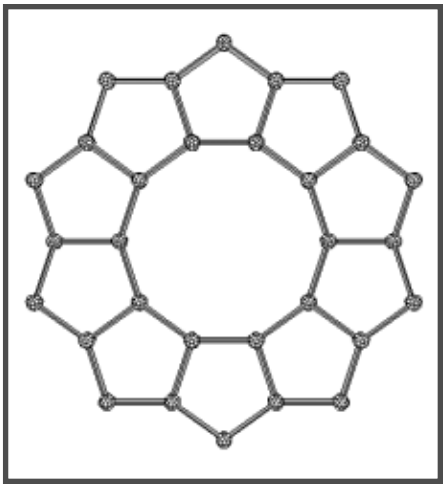
tilings with regular polygons, and of course, no gaps between the shapes. For triangles, squares, and hexagons, we see clearly that there are no gaps between shapes, and that a fixed number of the tiles fit evenly around a point with no gaps.

What about pentagons? Can we tile them with no gaps? What would be a good way to find out? Have the students build a regular pentagon, and attach more pentagons around the vertex. *How many pentagons can we fit around the vertex? Does it fit perfectly? Does this mean we cannot tile with them?* To discover what happens, have the teams continue building with pentagons. *How many ways can we tile with pentagons plus one other shape?* Which other shapes are needed to complete the tiling? Consider different ways to form a ring of pentagons with a gap in the middle. Several examples are shown in the figures.

In the different tilings of the teams, which ones have transitional symmetry? Which have rotational symmetry? Do any teams have tilings with no symmetry at all?

Conclude by having the students write their own definitions of non-periodic tilings in their math journals.





Assessment:

Study the tilings produced by the teams and review definitions in math journals. To meet the standard students must determine which shapes can be used to tile with regular pentagons. To exceed the standard their definitions must clearly distinguish between periodic and non-periodic tilings.

Standards Addressed:

* Mathematics standards addressing the study of the geometry of one, two, and three dimensions in a variety of situations (NCTM Standard 12).

Transfer Possibilities:

More advanced tilings in 2, and 3 dimensions ("Richert - Penrose Tilings," "3-D Triangles," "3-D Triangle Tiles," "Plato's Solids - I," and Plato's Solids - II".)