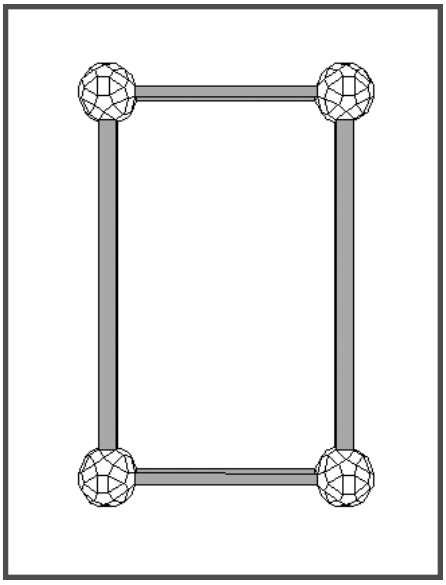
Zome System

Builds Genius!

Finding Tau

Mathematics
Intermediate Concept

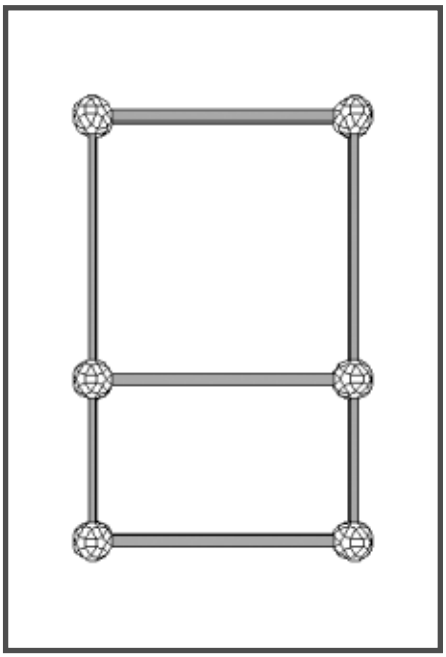


Lesson Objective:
Students find and explore the Tau notation, and start working on the uses and occurrences of the Golden Section in mathematics and nature.

Prerequisite Skills:
Knowledge of basic polygon shapes ("Geometric Shapes"). Previous work on irrational numbers like pi is helpful.

Time Needed:
One class period of 45 to 60 minutes.

Materials Needed:
- One or two Zome System Creator Kits for 25-30 students
- One metric ruler, or rulers with inches divided into tenths, per team
- One calculator per team



Procedure:
Divide the class into teams of 2-3 students, and distribute Zome System pieces, rulers, and calculators. Introduce the topic by asking a few students to draw a rectangle on the board, and measure the height and width. Determine the ratio of the long side to the short side for each rectangle. Compare each ratio to 1.6, and explain that this is a number relationship known as the Golden Section, Golden Ratio, or Divine Proportion.

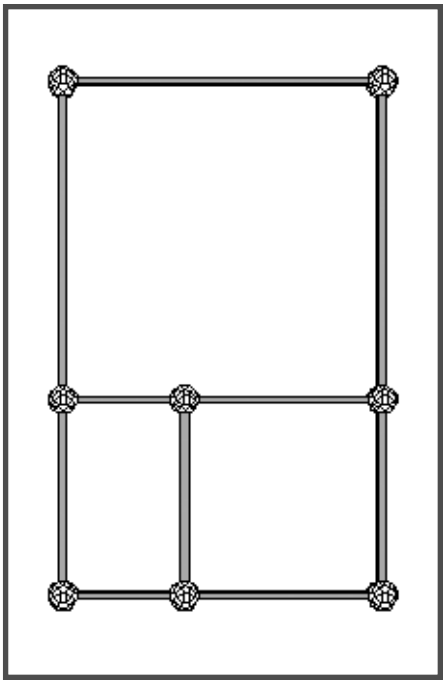
Their first task is to build a rectangle of the 2 short and 2 medium blue struts. *Is this a short, fat rectangle, or is it long and skinny? Have you seen this specific rectangle before?* The two different sides of this rectangle are in a special ratio called the Golden Section, represented by τ (the Greek letter tau). *What is a ratio? How do we describe or measure a ratio?* The sides are in the ratio 1:τ, where the

Finding Tau

short side is 1 unit and the long side is τ units. *How can we find tau? What is the best way to measure this rectangle? Should the width of the nodes be included in the measurement?*

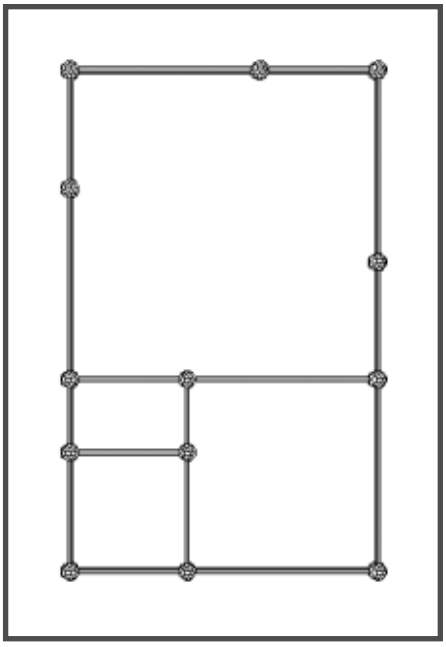
Ask the students to determine the ratio of the two sides of the rectangle. Representatives for a few teams can report their answer to the class. *Have all the teams got the same answer? What strategy did they use to determine the ratio?* To find the ratio, the teams should have divided the two edge lengths. For instance: 4.80/2.95 inches = 1.627.

In what unit is τ measured (a ratio does not have a unit)*?* Next, add a square to the rectangle, as shown in the graphic. Measure the new side, and write it down. *What is the ratio* (apx. 7.75 inches / 4.80 inches = 1.614)*?*



Record this ratio. *How close is it to the previous ratio?* This close ratio of sides tells us that our new rectangle is similar (same shape, different size) to the original one. Ask the teams to continue this process two more times, adding a square to the longer side of newly created rectangle. They should create a table to keep track of the ratios. *What is special with the Golden Rectangle? How do the rectangle and square interact with each other? What are the new lengths* (7.75, 12.55)*? What is the new ratio* (2.55 inches / 7.75 inches = 1.619)*? Can we continue adding squares? How?*

Bring the class back together and let them report their recorded ratios and any other findings. *How can we find an average ratio for all our measured ratios?*



The name of this ratio is tau, or τ. The actual value of tau is 1.6180339.... Like pi, the constant used in measuring circles and spheres, tau continues indefinitely, and its pattern doesn't repeat. This kind of number is called irrational. Each length we measured was longer than the one before it by 1.6180339... times. If our first length is 1 then the second is τ = 1.618.

Tau appears in countless places, usually as a ratio between two lengths. For example, most temples in ancient Greece, including the Parthenon in Athens, were built in a rectangular shape with the exact same proportion as the rectangle the students just made.

The tau ratio also occurs in many natural objects,

including the human hand. The next task is for the teams to determine the ratios between their finger bones. Using rulers, they should measure the length of the first section of their fingers (fingertip to first joint), and write down that measurement. Continue measuring each section of the finger, and then the large knuckle to the middle of the wrist (metacarpus). Make four columns on chart paper or the black board, labeled first, second, third, and fourth section. Have each student read off his or her finger measurements. Take the average for each column. The ratio of the second section to first section, the third to the second, and fourth to the third will all fall closely to tau.

Our navel also tends to divide our entire body into Golden Section, and our brows tend to divide our faces into the same ratio. The tau ratio can also be found in other growth patterns in nature. Old growth on many plants tend to be in Golden Proportion to new growth, etc.

As an extension activity students can be asked to build a structure of their own choosing, and the make copies of it in several Golden Section scales.

Assessment:
Study student's models, drawings, and notes in math journals. To meet the standard students must complete measurements of Zome System models and finger bones and derive ratios according to the activity in the lesson.

Standards Addressed:
* Mathematics standards addressing investigation of mathematical connections (NCTM Standard 4).
* Mathematics standards addressing the development of number and number relationships (NCTM Standard 5).
* Mathematics standards addressing the study of the geometry of one, two, and three dimensions in a variety of situations (NCTM Standard 12).
* Mathematics standards addressing extensive concrete experiences using measurement (NCTM Standard 13).

Transfer Possibilities:
More work on number patterns and the occurrence of the Golden Section in nature and the man-made world ("What is Reflection Symmetry?," "Multiple Reflection Symmetry," "Fun Fibonacci," "The Golden Thread-A History of Tau," and "Fibonacci Numbers and the Golden Section"). Doing algebra and trigonometry with the Golden Section ("Similarity and the Golden Section," and pages 21-24 in "Zome System Manual").