

Zome System

Builds Genius!

Fibonacci Numbers and the Golden Section

Mathematics Intermediate Concept

Lesson Objective:

Students will investigate the relationships between the additive properties of **Fibonacci numbers** and the additive properties of the **Golden Section**, and discover the connection between the two.

Prerequisite Skills:

Experience working with Fibonacci Sequences (“Multiple Reflection Symmetry,” “Fun Fibonacci,” and page 10 in Zome System Manual). Basic knowledge of the properties of the Golden Section (“Finding Tau,” page 20 in Zome System Manual, and “The Golden Thread - A History of Tau”).

Time Needed:

One or two class periods of 45-60 minutes

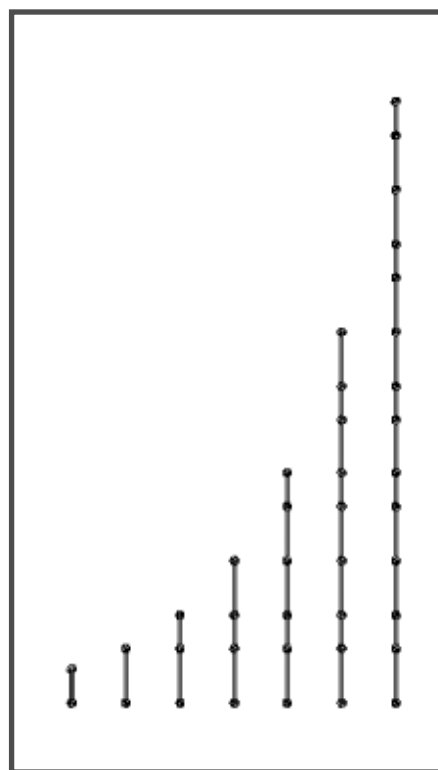
Materials Needed:

- Two Zome System Creator Kits for 25-30 students
- Graph paper
- Calculator (one per 5 students)

Procedure:

Divide the class into teams of 5 students, and distribute several Zome System nodes and struts of all three lengths in one color to each team. Review the concept of geometric progression from the class “Finding Tau”. In this progression of the golden section, two previous terms in the series add up to the next term in the series. *How can this be demonstrated in the lengths of the struts you have?*

Let the groups explore the struts for a few minutes to see if they can establish the progression in the lengths. Comparing the lengths of the longest strut with the short and medium added together can demonstrate the beginning of this series (short strut + medium strut = long



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strut). *What would be the next step in this series?* In the “Finding Tau” lesson, we learned that $X + 1 = X^2$ for the golden section, and that $t^2 = t + 1$.

Have the students continue the series by using the adding rule. *What is the pattern developing in this series?* The name of this pattern is the Fibonacci Sequence. *How do these numbers relate to the Golden Section? Is there any similarity to how they develop? How does this series differ from the Golden Section? Which one contains whole numbers? What kind of numbers does the Golden section contain?* If the two series are connected in the way they add the two previous values to obtain the third, there will be a connection between the ratios of adjacent numbers. Ask the students to see if they can discover what happens to the value of the ratios of adjacent numbers as the series grows, using graph paper and calculators. Again, allow a few minutes for the teams to explore the relationships on graphs.

The clearest way display the relationships is to draw a graph on the board and mark the value of t (1.6180339...) as a horizontal line above the x-axis. Ask students to plot the successive ratios of adjacent Fibonacci numbers, and mark them in the same graph. Thus the first ratio is $1/1=1$, the second is $2/1=2$, the third is $3/2=1.5$, the fourth is $5/3=1.666...$, $8/5=1.6$, $13/8=1.625$, etc. Students will see how the values alternates above and below the value of the Golden Section. The ratios rapidly converge on the exact value of t , and the connection between these series becomes clear.

The teams should both discuss mathematical and philosophical implications the their findings, and note their thoughts in their math journals.

Assessment:

Review the graphs created by the teams, as well as notes in individual math journals. To meet the standards, students must be able to show the similar additive property of the Golden Ratio lengths of the Zome System struts, and in the Fibonacci Sequence. They must also note that the Fibonacci Sequence and the Golden ratio differ in that the former consists of whole numbers, the latter is made up of irrational numbers. To exceed the standards their graphs must be plotted to show how successive ratios of adjacent Fibonacci numbers converge toward the Golden Section.

Standards Addressed:

- * Mathematics standards addressing **the development of number and number relationships** (NCTM Standard 5).
- * Mathematics standards addressing **number systems and number theory** (NCTM Standard 6).
- * Mathematics standards addressing the **exploration of patterns and function** (NCTM Standard 8).

Transfer Possibilities:

More advanced work with the Tau notation (pages 21-23 in Zome System Manual, and “Similarity and the Golden Section”).