Zome System

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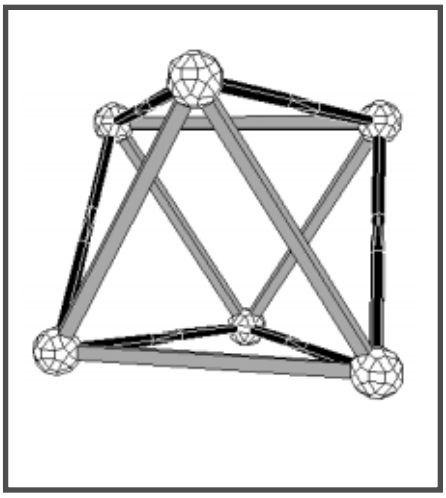
Euler's Formula for Polyhedra

Mathematics
Intermediate Concept

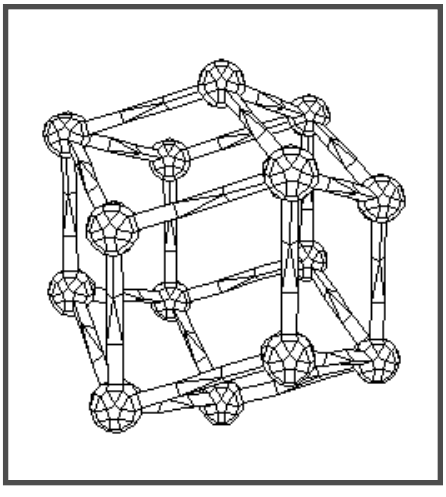
Lesson Objective:
Students will discover Euler's formula for polyhedra, and will be able to show that it works for any convex polyhedron.



Prerequisite Skills:
Ability to build and identify different polyhedra including the 5 platonic solids ("Naming 2D and 3D Shapes," "Plato's Solids - I," and "Plato's Solids - II").

Time Needed:
One or two class periods of 45-60 minutes.

Materials Needed:
- Two Zome System Creator Kits for 25-30 students
- A potato or a piece of Styrofoam
- Sharp knife



Procedure:
Start the lesson with a brief review of what your students know about polyhedra. *What is a polyhedron or a solid? How are they named? Can anyone name the Platonic solids? How many are there? Who can name any other solids?*

List on the board the solids from the table below. Propose to the students that there is a numerical relationship between the faces, edges, and vertices of any given polyhedron. *How could we go about finding this relationship?* Divide the class into teams of three to five students. Their challenge is to find the general relationship formula. The teams should start by building one of each of the models from the list on the board. They should copy the names of all the listed solids into their math journals and create a table for the number of faces, edges, and vertices, using the shapes they have built to count each feature.

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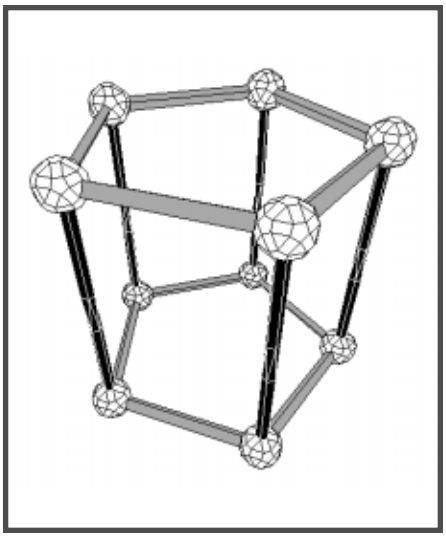
When completed their tables will look as follows:

	Faces	Edges	Vertices
Tetrahedron	4	6	4
Octahedron	8	12	6
Hexahedron (Cube)	6	12	8
Icosahedron	20	30	12
Dodecahedron	12	30	20
Triangular Prism	5	9	6
Pentagonal Prism	7	15	10
Pentagonal Pyramid	6	10	6



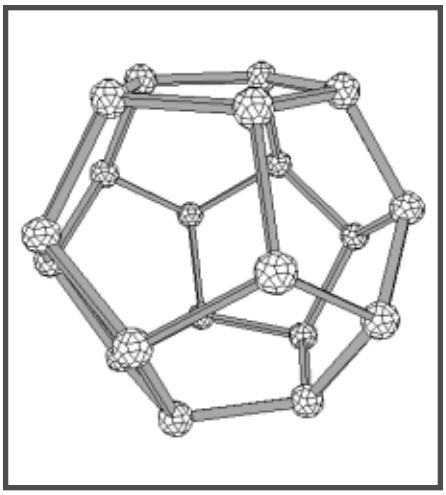
The students should continue their discussions until they find a pattern in these numbers in the form of a relationship between the edges, faces, and vertices of each shape. Try adding and subtracting the numbers in various combinations until they find a formula which gives the same answer in every case.

Once they derive the correct formula, write it on the board:

Faces + Corners = Edges + 2

F + C = E + 2

or, F + C - E = 2



This formula is named "Euler's Formula" (pronounced "oiler") after the Swiss mathematician Leonhard Euler who discovered this relationship in 1752. Euler showed that his formula works for <u>any</u> convex polyhedron, whether it is regular or irregular.

If time allows, perform a demonstration of this using a potato (or a piece of Styrofoam) and a sharp knife. Cut slices from the potato until a random flat-faced polyhedra is produced. Count the faces, edges, and vertices, using a marker to keep track of the count. Demonstrate on the board that the totals fit into Euler's Formula. Alternatively, you could let the students build a number of irregular polyhedra and check that the formula holds up.

Can anyone think of a way this formula might be useful? For example, if a builder knows the number of bars and connectors in a dome, he or she can predict the number of

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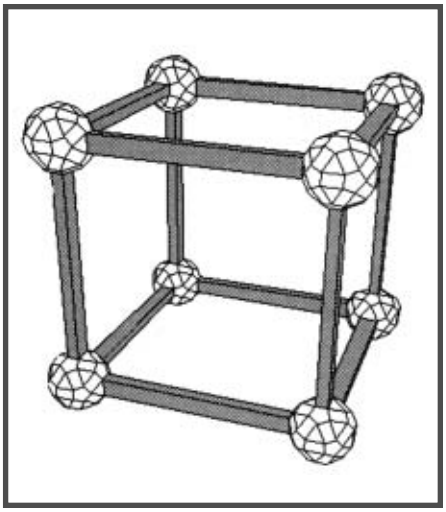
panels required to build the dome.

Assessment:
Review tables and formulas in student math journals.
To meet the standard students must build and name the listed polyhedra, count their components, and attempt to construct a general formula. To exceed the standard they must correctly verbalize Euler's formula.



Standards Addressed:
* Mathematics standards addressing mathematical problem solving as a method of inquiry and application (NCTM Standard 1).
* Mathematics standards addressing mathematics as a means of communications (NCTM Standard 2).
* Mathematics standards addressing mathematics as reasoning (NCTM Standard 3).
* Mathematics standards addressing the study of the geometry of one, two, and three dimensions in a variety of situations (NCTM Standard 12).

Transfer Possibilities:
Further exploration of polyhedra geometry ("Archimedian Solids," and constructions 4, 5, 6, and 8 in Zome System Manual). Discussions of constructing proofs for mathematical formulas.