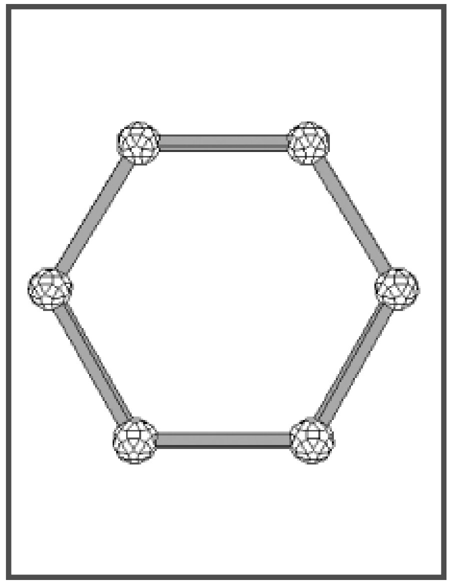
Zome System

Builds Genius!

Beehive City

Mathematics
Advanced Concept

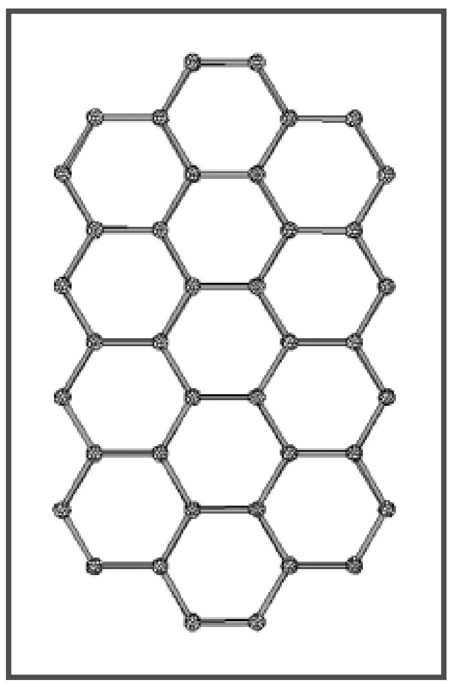


Lesson Objective:
Students will discover why the shape of the bee's honeycomb is derived directly from the geometry of densest packing of spheres in three dimensions.

Prerequisite Skills:
Knowledge of basic polygons ("Geometric Shapes"), and ability to define a two-dimensional versus a three-dimensional figure ("2-D and 3-D Shapes"). Ability to build and name polyhedra shapes ("Plato's Solids - I," "Plato's Solids - II," and "Naming 2-D and 3-D shapes").

Time Needed:
Two class periods of 45 to 60 minutes

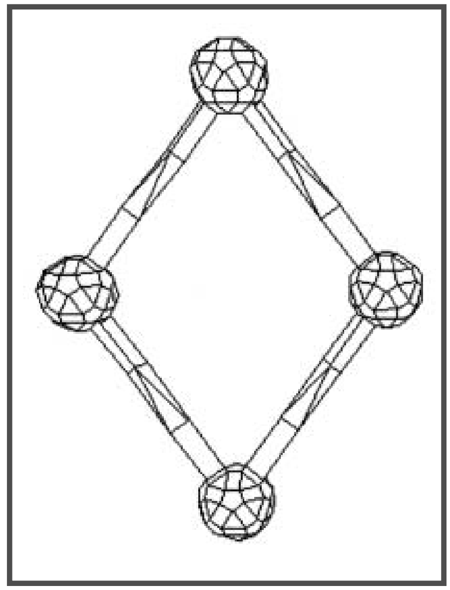
Materials Needed:
- Three Creator Kits or 2 Creator Kits plus two bags of yellow struts
- Pennies, about 50 cents per team
- Ping Pong balls, about 10 per team
- Card stock grids of diamonds to cut out, see Resources section in back of this document
- Scissors, tape, about 1 set per team
- All-purpose glue (one container per team)
- Real bee honeycomb (one for each team if possible)



Procedure:
Begin by inviting the students to discuss honeycombs. *In what shapes do bees make their honeycombs? Does anyone know why all bees have honeycombs in this shape?* Lead a discussion on the efficiency of nature and its structures. *Is there any example where nature wastes energy in accomplishing a structure, pathway, or task?* Let students list as many examples they can think of in which nature conserves energy and maximizes efficiency. If no already listed, you might suggest:

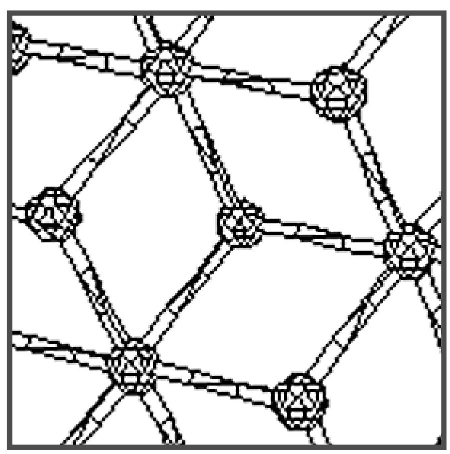
surfaces in bubbles, packaging of seeds, dormancy in winter, flow of a river, crystals in rocks and minerals, and the large ears on rabbits as temperature controls.

What is the purpose of the honeycomb? The purpose is dual; the honeycomb serves as an incubation chamber for growing worker bees, as well as to store honey, which is their food for the winter. So, it makes sense that they would want to produce the most storage chambers in the given amount of space.



What sort of spacing would provide this? How many other cells should each cell touch? What would be a good method of discovering the spacing? Distribute the pennies to the teams, and let them experiment with different spacing combinations. *What is the most number that can touch each other without overlapping? What is the least? In which configuration can the most pennies fit into a space, without overlapping? How can we draw lines to get rid of the gaps in between the pennies?* Once the teams have found the most effective grid structure, they should build a copy using blue Zome System struts. This is called a hexagonal packing structure.

Bees live in three dimensions. Using the ping pong balls and glue, repeat the exercises. *In what way can you get each ball to touch the least other balls? How about touching the most?* Moving in three dimensions, count the balls that touch (nearest neighbors). One way to accomplish this is to start with one ball, and see how many spheres can be glued to that central sphere. Another way is to glue together the closest packing formation in two dimensions, stack a closest packing layer on top of that and glue it, and when the glue is dry, glue another layer below that. Each team should have created the closest packing of spheres.

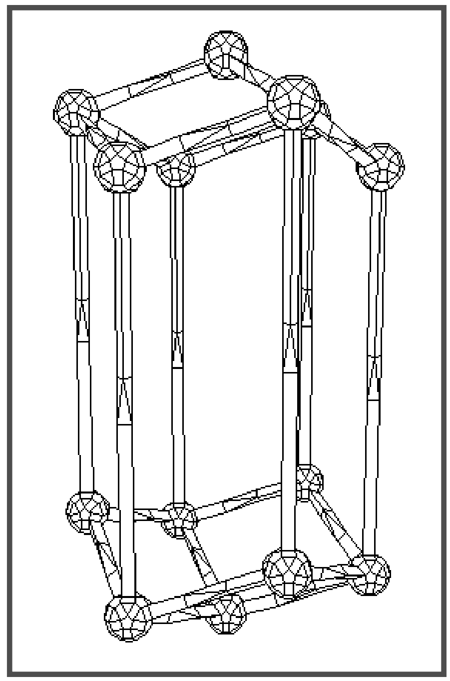


How can we apply the hexagonal structure from the penny model to the three dimensional version? Instead of passing lines through the points of contact between the pennies, pass a plane through the points of contact between ping pong balls.

What would this look like? Can you describe the shape that would be formed? How can we observe this? If the class cannot visualize the shapes, have them use the cardstock with diamonds copied onto it. Cut out the shapes, and fit them together. This shape would have 12 identical rhombic faces. It is called a rhombic dodecahedron. This shape can fill space with copies of itself. Challenge each group to build a rhombic dodecahedron in Zome System. *What connection does this have with the honeycomb?*

Divide the honeycomb into pieces, one for each group, or distribute the pieces. *What does the bottom surface of the comb look like? What is on the other side? Is a honeycomb made up of just hexagons?* Discuss how the bottom of the comb is divided into 3 equal diamonds, which come together at angles. Compare it with the rhombic dodecahedron. *What is similar between the two?* This diamond has diagonals in a ratio of $1:\sqrt{2}$. *How can we make a complete Zome System model of a bee honeycomb? How can we change the rhombic dodecahedron to make it resemble a honeycomb?* Give the teams time to find that they must use short and long yellow sticks to build elongated rhombic dodecahedrons to form a long hexagonal column with triads of diamonds. They must remove the triad on one of the ends. Several of these columns can then be packed together to form a honeycomb.

How should we build the other side of this? Look at the real honeycomb to see how two columns, one form each side, fit together base to base. *How would we go about fitting in the other side of chambers? How does the position of columns on one side compare to the position on the other side? How do they relate? How many bee chambers fit into a cubic inch of real honeycomb?* Complete the lesson with a short review of the effectiveness of the honeycomb as a packing system in three dimensions.



<u>Assessment:</u>
Question teams as they work, and review their math journals. To meet the standard, students must determine which shapes provide the most effective packing in two and three dimensions. To exceed the standard they must build a Zome System honeycomb and verbalize a rule of why this structure meets the challenge fazed by the bees.

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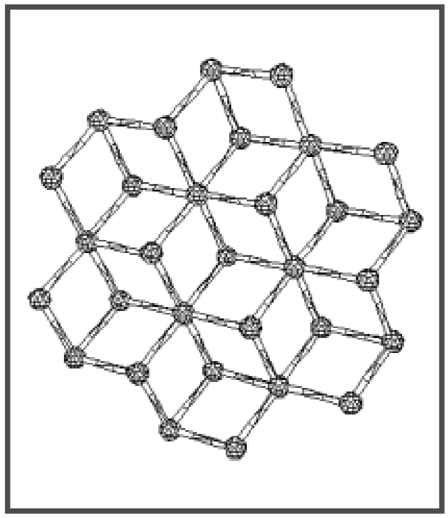
Standards Addressed:

* Science standards relating to biological structures, energy use in nature, and animal behavior.
* Mathematics standards addressing the extension of problem solving (NCTM Standard 1).
* Mathematics standards addressing the continued study of the geometry of two and three dimensions (NCTM Standard 7).



Transfer Possibilities:

Continued exploration of Polyhedra shapes ("Archimedian Solids", and constructions 4, 5, 6, and 8 in Zome System Manual).