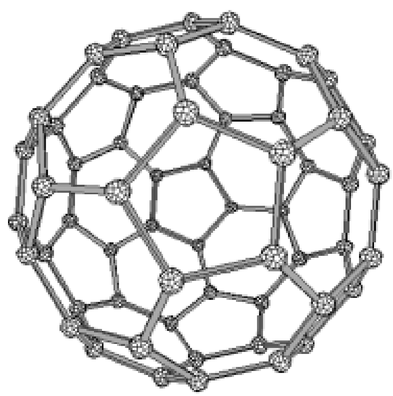
Mathematics / Philosophy
Advanced Concept

Lesson Objective:
Students will explore semi-regular, or Archimedian solids. They will determine how many such solids exist, which of them can be built with Zome System, and they will build one of them in its entirety.



Prerequisite Skills:
Knowledge of basic polygons ("Geometric Shapes"), and ability to define a two-dimensional versus a three-dimensional figure ("2-D and 3-D Shapes"). Experience building geometric solids ("Plato's Solids - I," "Plato's Solids - II," and "2-D and 3-D Stars").

Time Needed:
Two class periods of 45 to 60 minutes.

Materials Needed:
- Two or three Zome System Creator Kits for 25-30 students, or two kits and extra sets of blue struts.
- Four to six packs of supplementary green Zome System struts, if available
- Card stock polygons from Resource section of this document
- One pair of scissors per team
- One roll of tape per team

Procedure:
Prepare for the class by producing a large number of regular paper polygons. An easy way to do this is to enlarge the shapes in the resource section 200-300%, and cut them out. The paper polygons will be easiest to work with if they are copied onto, or glued onto, slightly heavier paper or card stock.

Start the class with a brief review of polyhedra. *Are polyhedra always made up of polygons? How are polyhedra named?*

Face angles of polygons

triangle	60°
square	90°
pentagon	108°
hexagon	120°
octagon	135°
decagon	144°

Archimedian Solids

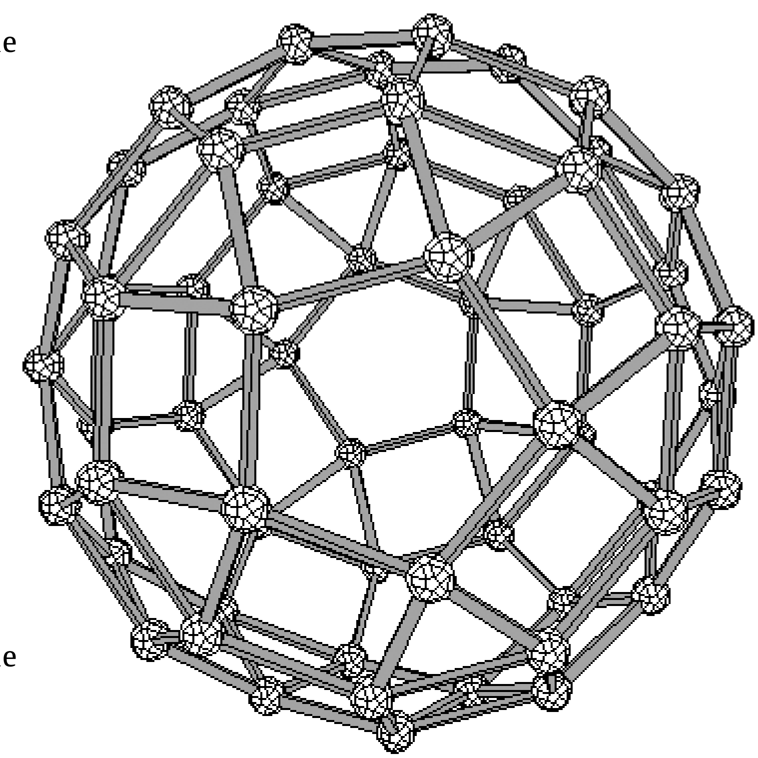
Zome System

Builds Genius!

What is a regular polyhedron, or Platonic solid (convex figures where all faces, edge lengths, and angles are identical)*? How many such shapes exist* (5)*? Which polygons are they made up of? What are they called?* In this lesson the students will explore another family of polyhedra called semi-regular, or Archimedian solids. *Why would a polyhedron be called semi-regular* (they are made up of more than one type of regular polygons and have identical vertices)*?*

Divide the class into teams of 3-4 students and distribute Zome System pieces, the scissors, tape, and card stock polygons. The task for the teams is to discover how many semi-regular polyhedra exist using paper polygons and Zome System. *How should we go about determining how such solids exist? Is the number unlimited or are there a set number?* Ask the students to write down their predictions in their math journals. Discuss the various strategies the students propose.

Vertices buildable with just blue struts
1 pentagon, 2 hexagons
2 pentagons, 2 triangles
2 decagons, 1 triangle
2 pentagons, 2 triangles
1 pentagon, 2 squares, 1 triangle

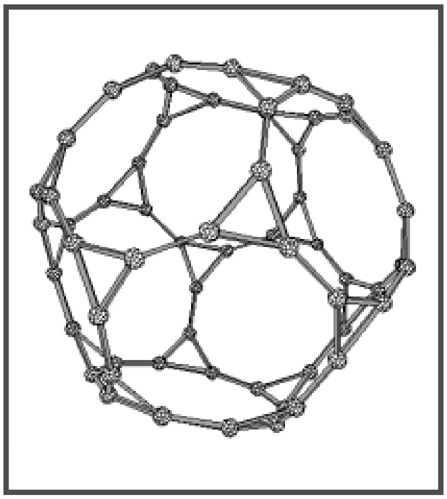


Vertices requiring green struts
2 hexagons, 1 triangle
2 squares, 2 triangles
3 squares, 1 triangle
2 hexagons, 1 square
1 octagon, 1 square, 1 hexagon
1 pentagon, 4 triangles

Vertices not buildable in Zome System
1 square, 4 triangles
1 pentagon, 4 triangles

One strategy is to first decide how many different ways polygons can be fitted together to create a polyhedron vertex.

Archimedian Solids



Students should keep in mind that vertices made up of identical polygons will build the 5 Platonic, or regular solids. To be a “legitimate” vertex the sum of the meeting face angles must be less than 360°. The student teams should build as many different vertices as they can find using the Zome System elements. They can supplement by also taping together cardstockr polygons. Allow 25-30 minutes for this exploration. The student teams must make notes of all their findings in their math journals.

Go through the produced vertices with the whole class. *How many of the 13 possible vertices did the teams find? How many could be built with blue struts? How many reqire the green struts?* The 13 vertices are shown in the table on the right. Six of the vertices, and thus the solids, require the supplementary green Zome System struts. Two of the solids cannot be built with Zome System, and must be modeled with cardstock polygons.

The next task for the teams is to choose one of the vertices that can be built in Zome System, and make a model of the complete solid. They should learn the name and composition of this solid and introduce it to the class. The table below

Name of Solid	Faces
Cuboctahedron:	6 squares and 8 triangles
Great rhombicosadodecahedron	12 decagons, 20 hexagons, 30 squares
Great rhombicuboctahedron	6 octagons, 8 hexagons, 12 squares
Icosadodecahedron	12 pentagons, 20 triangles
Small rhombicosadodecahedron	12 pentagons, 20 triangles, 30 squares
Small rhombicuboctahedron	18 squares, 8 triangles
Snub cube	6 squares, 32 triangles
Snub dodecahedron	12 pentagons, 80 triangles
Truncated cube	6 octagons, 8 triangles
Truncated dodecahedron	12 decagons, 20 triangles
Truncated icosahedron	12 pentagons, 20 hexagons
Truncated octahedron	6 squares, 8 hexagons
Truncated tetrahedron	4 hexagons, 4 triangles

can be written on the board, or given as a handout to the students. *What can we notice about the number of the different polygons in the solids? Which numbers are repeated? How do the numbers of different polygons relate to the regular, Platonic, solids?*

The solids listed were first described by the Greek mathematician and philosopher Archimedes (c. 287-212 BC). The Archimedian solids are not common in nature. However, the truncated icosahedron is the shape of the C_{60} carbon molecule or "Buckyball". Robert Curl and Richard Smalley of Rice University in Texas, and Harold Kroto of Sussex University in England, were awarded the 1996 Nobel Prize in chemistry for the discovery of this molecule.

An interesting extension activity is to test Euler's formula against the models built by the students.

Assessment:

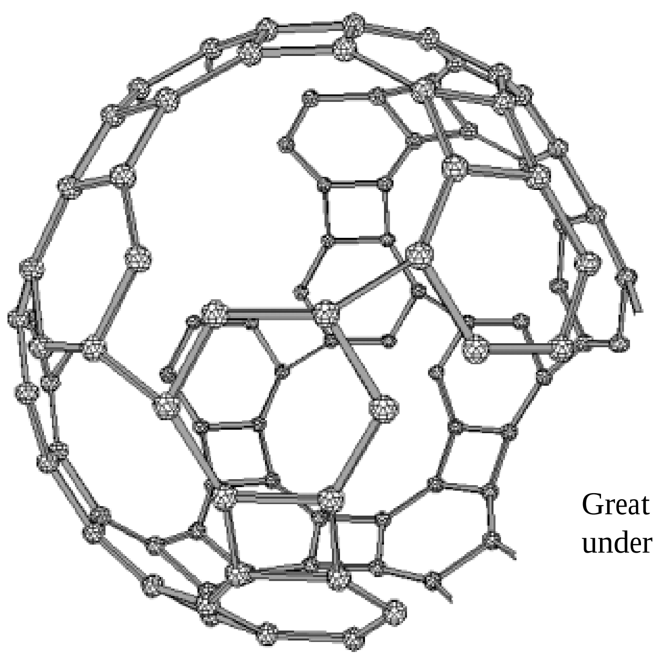
Observe and students as they build their structures, and take notes of their findings. Review notes in math journals. To meet the standard, students must complete at least 5 of the vertices using eithe Zome System elements or cardstock polygons. To exceed the standard they must identify all 13 possibilities, and determine how many can be built with Zome System.

Standards Addressed:

* Mathematics standards addressing the study of the geometry of one, two, and three dimensions in a variety of situations (NCTM Standard 12).

Transfer Possibilities:

Continued exploration of Polyhedra shapes (constructions 4, 5, 6, and 8 in Zome System Manual). More work on three-dimensional tessellations ("3-D Triangle Tiles" and "Beehive City").



Great Rhombicosadodecahedron under construction.